

nag_kelvin_ker (s19acc)**1. Purpose**

nag_kelvin_ker (s19acc) returns a value for the Kelvin function $\ker x$.

2. Specification

```
#include <nag.h>
#include <nags.h>
```

```
double nag_kelvin_ker(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the Kelvin function $\ker x$.

The function is based on several Chebyshev expansions.

For large x , $\ker x$ is so small that it cannot be computed without underflow and the function evaluation fails.

4. Parameters

x

Input: the argument x of the function.
Constraint: $x > 0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings**NE_REAL_ARG_GT**

On entry, **x** must not be greater than $\langle value \rangle$: $x = \langle value \rangle$.
x is too large, the result underflows and the function returns zero.

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to 0.0: $x = \langle value \rangle$.
The function is undefined and returns zero.

6. Further Comments

Underflow may occur for a few values of x close to the zeros of $\ker x$, which causes a failure **NE_REAL_ARG_GT**.

6.1. Accuracy

Let E be the absolute error in the result, ϵ be the relative error in the result and δ be the relative error in the argument. If δ is somewhat larger than the **machine precision**, then we have $E \simeq |x(\ker_1 x + \operatorname{kei}_1 x)/\sqrt{2}| \delta$, $\epsilon \simeq |x(\ker_1 x + \operatorname{kei}_1 x)/\sqrt{2} \ker x| \delta$.

For very small x , the relative error amplification factor is approximately given by $1/|\log x|$, which implies a strong attenuation of relative error. However, ϵ in general cannot be less than the **machine precision**.

For small x , errors are damped by the function and hence are limited by the **machine precision**.

For medium and large x , the error behaviour, like the function itself, is oscillatory, and hence only the absolute accuracy for the function can be maintained. For this range of x , the amplitude of the absolute error decays like $\sqrt{\pi x/2} e^{-x/\sqrt{2}}$ which implies a strong attenuation of error. Eventually, $\ker x$, which asymptotically behaves like $\sqrt{\pi/2} x e^{-x/\sqrt{2}}$, becomes so small that it cannot be calculated without causing underflow, and the function returns zero. Note that for large x the errors are dominated by those of the **math library** function `exp`.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9.9 p 379.

7. See Also

nag_kelvin_ber (s19aac)
 nag_kelvin_bei (s19abc)
 nag_kelvin_kei (s19adc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_kelvin_ker(s19acc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
    double x, y;

    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vprintf("s19acc Example Program Results\\n");
    Vprintf("      x      y\\n");
    while (scanf("%lf", &x) != EOF)
    {
        y = s19acc(x, NAGERR_DEFAULT);
        Vprintf("%12.3e%12.3e\\n", x, y);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s19acc Example Program Data
      0.1
      1.0
      2.5
      5.0
     10.0
     15.0
```

8.3. Program Results

```
s19acc Example Program Results
      x      y
1.000e-01  2.420e+00
1.000e+00  2.867e-01
2.500e+00 -6.969e-02
5.000e+00 -1.151e-02
1.000e+01  1.295e-04
1.500e+01 -1.514e-08
```